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THE OPTIMAL INFLATION RATE REVISITED

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The optimal inflation rate revisited

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Abstract

We challenge the widely held belief that leading theories of monetary non-neutrality cannot predict an optimal positive inflation rate. In fact we find that for plausible calibrations of our model, optimal trend inflation is justified by the Phelps argument that the inflation tax should be part of an optimal (distortionary) taxation scheme.

Jel codes: E52, E58, J51, E24.
Keywords: trend inflation, monetary and fiscal policy, Ramsey plan.

1 Introduction.

Optimal monetary policy analyses (Khan *et al.*, 2003; Schmitt-Grohé and Uribe, SGU henceforth, 2004a) identify two key frictions driving the optimal level of long-run (or trend) inflation. The first one is the adjustment cost of goods prices, which invariably drives the optimal inflation rate to zero. The second one is the existence of monetary transaction costs that arise unless the central bank implements the Friedman rule, i.e. a negative steady-state inflation rate as long as the steady-state real interest rate is positive.

Phelps (1973) conjectured that to alleviate the burden of distortionary taxation it might be optimal for governments to resort to monetary financing, driving a wedge between the private and the social cost of money. SGU (2004a) show that, even accounting for the Phelps' effect, the optimal inflation rate lies between zero and the Friedman rule, being very close to zero for apparently plausible parameterizations of the model. A consensus seems to exist that monetary transactions costs are relatively low at zero inflation, and that stable prices are the proper policy target. In their survey of the literature, SGU (2010) argue that the optimality of zero inflation is robust to other frictions, such as nominal wage adjustment costs, downward wage rigidity, hedonic prices, incompleteness of the tax system, the zero bound on the nominal interest rate.

This theoretical result is in sharp contrast with empirical evidence. For instance, both in the US and in the Euro area, average inflation rates over the 1970-1999 period have been close to 5%. Further, even the widespread central bank practice of adopting inflation targets between 2 and 4% is apparently at odds with theories of the optimal inflation rate (SGU, 2010).

In this paper we reconsider the issue, showing that dismissal of the Phelps' effect is due to an unrealistic parameterization for public expenditures and overall taxation and thus appears premature. In the literature, standard calibrations of public expenditures focus on public consumption-to-GDP ratios, typically set at 20% (SGU, 2004a; Aruoba and Schorfeide, 2009). This follows a long-standing tradition in business cycle models, where only public consumption decisions have real effects. In our framework this choice is not correct, because the focus here is on distortionary financing of public expenditures in steady state, where also other components of public expenditure matter. As a matter of fact, public consumption accounts for a limited component of the overall fiscal burden in OECD countries (Table 1).

Table 1 – Government expenditures and revenues (1998-2008)*

	(1)	(2)	(3)		(1)	(2)	(3)
Australia	18,00	16,97	36,26	Japan	17,07	21,28	31,81
Austria	19,10	32,29	49,71	Netherlands	23,57	22,19	45,34
Belgium	22,13	27,82	49,39	New Zealand	17,97	20,89	42,01
Canada	19,49	21,56	42,08	Norway	20,76	23,54	56,63
Czech Republic	21,24	22,81	40,12	Poland	17,95	25,34	39,20
Denmark	25,84	27,88	55,96	Portugal	19,57	25,48	41,59
Finland	21,75	27,74	53,12	Slovak Republic	20,24	21,35	36,55
France	23,39	29,21	49,90	Spain	17,75	21,52	38,67
Germany	18,96	27,58	44,61	Sweden	26,67	29,03	57,21
Greece	16,52	28,32	40,19	Switzerland	11,4	23,48	34,40
Hungary	21,98	27,42	43,20	United Kingdom	19,83	22,28	40,38
Ireland	15,11	19,40	44,16	United States	15,26	20,51	33,47
Italy	19,10	28,94	45,25	Euro area	20,17	27,11	45,39

(1) public consumption; (2) other public expenditures; (3) total revenues
* ratios to GDP – Source OECD

Even if a proportion of total expenditures goes into production subsidies, it is apparent that distortionary taxation substantially exceeds public consumption, in order to finance redistributive policies. For instance in the US, according to the National Accounts (NIPA) dataset, in this period government transfer payments and government purchases respectively were 11.8% and almost 20% of GDP.

We show that just allowing for a plausible parameterization of public transfers to households in the SGU (2004a) model reverses their conclusion about the optimal inflation rate, which now monotonically increases from 2% to 12%

as the transfers-to-GDP ratio goes from 10% to 20%. We also find that an identical increase in the public-consumption-to GDP ratio would have a negligible impact on the optimal inflation rate. So, what is special about public transfers? To grasp the intuition behind our result, assume that lump-sum taxes can be used to finance expenditures. In case of public transfers the overall effect on the household budget constraint is nil, and labor-consumption decisions are unchanged. By contrast, an increase in public consumption generates a negative wealth effect that raises the labor supply. If lump sum taxes are not available, the different wealth effect, i.e. the different labor supply response, explains why financing transfers requires higher tax rates than financing an identical amount of public consumption. Since the incentive to monetary financing is increasing in the amount of tax distortions, this explains why the optimal financing mix requires stronger reliance on inflation when we take transfers into account. Our result is robust to the inclusion of nominal wage rigidity, and is strengthened when we allow for a moderate degree of price and wage indexation (20%).

Another contribution of the paper is the introduction of consumption scale effects in the monetary transactions technology, in line with existing theoretical models (Baumol, 1952; Khan *et al.*, 2003) and with empirical evidence (Attanasio *et al.*, 2002). We find that such consumption scale effects unambiguously contribute to raise the optimal inflation rate. The intuition behind this result is simple. An increase in inflation allows a reduction in distortionary taxation but it raises the monetary transaction costs. This latter effect is weakened when the transaction cost is inversely related to the amount of consumption, which, in turn, increases if the tax rate falls.

Finally, we make a preliminary attempt to test the empirical plausibility of our results, calibrating the model to the US economy. Our purpose is to benchmark the optimal inflation rate against Fernandez-Villaverde and Rubio-Ramirez (2008) estimates of the time-varying inflation target implicitly adopted by the Federal Reserve over the period 1957-2000 and over the high inflation subsample 1973-1991. We consider different estimates of price rigidities found in the literature, and find that in all cases the optimal inflation rate is positive and increasing during the 1973-1991 subsample.

The remainder of the paper is organized as follows. The next section introduces the model. Section 3 defines the competitive equilibrium. Section 4 illustrates our main results. Section 5 considers the consumption scale effects on the transaction costs. In section 6 we outline a calibration of our model to the US economy. Section 7 concludes.

2 The model.

We consider a simple infinite-horizon production economy populated by a continuum of households and firms whose total measures are normalized to one. Monopolistic competition and nominal rigidities characterize both product and labor markets. A demand for money is motivated by assuming that money facilitates transactions. The government finances an exogenous stream of purchases by levying distortionary income taxes and printing money. Optimal policy is set according to a Ramsey plan.

2.1 Households

The representative household (i) maximizes the following utility function

$$U = \sum_{t=0}^{\infty} \beta^t u\left(C_{t,i}, l_{t,i}\right); \; u\left(C_{t,i}, l_{t,i}\right) = \ln C_{t,i} + \eta \ln\left(1 - l_{t,i}\right) \tag{1}$$

where $\beta \in (0,1)$ is the intertemporal discount rate, $C_{t,i} = \left(\int_0^1 c_{t,i}(j)^{\rho} dj\right)^{\frac{1}{\rho}}$ is a consumption bundle, $l_{t,i}$ is a differentiated labor type that is supplied to all firms. The price index associated to the consumption bundle is $P_t = \left(\int_0^1 p_t(i)^{\frac{\rho}{\rho-1}} di\right)^{\frac{\rho-1}{\rho}}$.

The flow budget constraint in period t is given by

$$C_{t,i}\left(1 + S_{t,i}\right) + \frac{M_{t,i}}{P_{t,i}} + \frac{B_{t,i}}{P_t} = \frac{\left(1 - \tau_t\right) w_{t,i} l_{t,i}}{P_t} + \frac{M_{t-1,i}}{P_t} + \frac{T_t}{P_t} + \frac{R_{t-1} B_{t-1,i}}{P_t} \tag{2}$$

where $w_{t,i}$ is the nominal wage; τ_t is the labor income tax rate; T_t is a lump-sum transfer from central bank profits; θ_t denotes firms profits; R_t is the gross nominal interest rate, $B_{t,i}$ is a riskless bond that pays one unit of currency in period $t+1$. $M_{t,i}$ defines nominal money holdings to be used in period $t+1$ in order to facilitate consumption purchases.

Consumption purchases are subject to a transactions cost

$$S_{t,i} = s(v_{t,i}), \quad s'(v_{t,i}) > 0 \tag{3}$$

where $v_{t,i} = \left(\frac{P_{t,i} C_{t,i}}{M_{t,i}}\right)$ is the household's consumption-based money velocity. The features of $s(v_{t,i})$ are such that a satiation level of money $v^* > 0$ exists where the transaction cost vanishes and, simultaneously, a finite demand for money is associated to a zero nominal interest rate. Following SGU (2004a) the transaction cost is parameterized as

$$s(v_{t,i}) = A v_{t,i} + \frac{B}{v_{t,i}} - 2\sqrt{AB} \tag{4}$$

2.1.1 Consumption and money demand decisions

The first-order conditions of the household's maximization problem are:[1]

$$c_t(j) = C_t \left(\frac{p_t(j)}{P_t}\right)^{\frac{1}{\rho-1}} \tag{5}$$

[1] When solving its optimization problem, the household takes as given goods and bond prices. As usual, we also assume that the household is subject to a solvency constraint that prevents him from engaging in Ponzi schemes.

$$\lambda_t = \frac{u_c\left(C_t, l_t\right)}{1 + s(v_t) + v_t s'(v_t)} \tag{6}$$

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta R_t \frac{P_t}{P_{t+1}} \tag{7}$$

$$\frac{R_t - 1}{R_t} = s'(v_t) v_t^2 \tag{8}$$

Equation (5) is the demand for the good j. As in SGU (2004a) condition (6) states that the transaction cost introduces a wedge between the marginal utility of consumption and the marginal utility of wealth, that vanishes only if $v = v^*$. Equation (7) is a standard Euler condition. Equation (8) implicitly defines the household's money demand function.

2.2 Firms' pricing decisions

Each firm (j) produces a differentiated good using the production function:[2]

$$y_t(j) = z_t l_{t,j}, \tag{9}$$

where z_t denotes a productivity shock[3] and $l_{t,j}$ is a standard labor bundle:

$$l_{t,j} = \left[\int_0^1 l_{t,j}(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}} \tag{10}$$

Firm (j) demand for labor type (i) is

$$l_{t,j}\left(i\right) = \left(\frac{w_{t,i}}{W_t}\right)^{-\sigma} l_{t,j} \tag{11}$$

where $W_t = \left[\int_0^1 w_{t,i}^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}$ is the wage index.

We assume a sticky price specification based on Rotemberg (1982) quadratic cost of nominal price adjustment:

$$\frac{\xi_p}{2}\left(\frac{P_t(j)/P_{t-1}(j)}{\pi_{t-1}^{\delta}} - 1\right)^2 \tag{12}$$

where $\xi_p > 0$ is a measure of price stickiness and $\pi_t = P_t/P_{t-1}$ denotes the gross inflation rate and $\delta \in [0,1]$ is the degree of price indexation to past inflation.

[2] We abstract from capital accumulation and assume constant returns to the scale of employed labor. The consequences of these two assumptions are discussed in SGU (2006) and SGU (2010) respectively. Our results are not affected by the introduction of diminishing return of scale for labor (simulation results available upon request).

[3] We assume that $\ln z_t$ follows an $AR(1)$ process.

In a symmetrical equilibrium the price adjustment rule satisfies:

$$\frac{z_t l_t (\rho - mc_t)}{1-\rho} + \xi_p \frac{\pi_t}{\pi_{t-1}^{\delta_p}} \left(\frac{\pi_t}{\pi_{t-1}^{\delta_p}} - 1 \right) = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \xi_p \left[\frac{\pi_{t+1}}{\pi_t^{\delta_p}} \left(\frac{\pi_{t+1}}{\pi_t^{\delta_p}} - 1 \right) \right] \quad (13)$$

where

$$mc_t = \frac{1}{az_t} \frac{W_t}{P_t}$$

From (5) it would be straightforward to show that $\frac{1}{\rho} = \mu^p$ defines the price markup that obtains under flexible prices.

2.3 Wage-setting decisions

The labour market is also characterized by monopolistic competition and rigid nominal wages. Under flexible wages

$$\frac{W_t}{P_t} = \mu^w \Omega_t \frac{u_l (C_t, l_t)}{u_c (C_t, l_t)} \quad (14)$$

where $\mu^w = \sigma (\sigma - 1)^{-1}$ denotes the gross wage markup and $\Omega_t = \frac{1+s(v_t)+v_t s'(v_t)}{1-\tau_t}$ denotes the policy wedge, which depends on both tax and inflation decisions.

We model nominal wage stickiness as in Rotemberg (1982). Each household maximizes the expected value of equation (1) subject to the (2), (11) and to

$$\frac{\xi_w}{2} \left(\frac{W_t(j)/W_{t-1}(j)}{\pi_{t-1}^{\delta_w}} - 1 \right)^2 \quad (15)$$

where $\xi_w > 0$ is a measure of wage stickiness and $\delta_w \in [0, 1]$ is the degree of wage indexation to past inflation.

As result, in a symmetrical equilibrium, the wage adjustment rule satisfies:

$$\left[(1-\tau_t) \frac{W_t}{P_t} + \frac{\mu^w u_l (C_t, l_t) (1 + s(v_t) + v_t s'(v_t))}{u_c (C_t, l_t)} \right] \frac{l_t}{\mu^w - 1} +$$
$$+ \xi_w \left[\frac{\omega_t}{\pi_{t-1}^{\delta_w}} \left(\frac{\omega_t}{\pi_{t-1}^{\delta_w}} - 1 \right) \right] = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \xi_w \left[\frac{\omega_{t+1}}{\pi_t^{\delta_w}} \left(\frac{\omega_{t+1}}{\pi_t^{\delta_w}} - 1 \right) \right] \quad (16)$$

where $\omega_t = \frac{W_t}{W_{t-1}}$.

2.4 The government

The government supplies an exogenous, stochastic and unproductive amount of public good G_t and implements redistributive policies through transfers T_t. Government financing is obtained through a labor-income tax, money creation and issuance of one-period, risk free (non-contingent) nominal obligations. The government's flow budget constraint is then given by[4]

$$R_{t-1}\frac{B_{t-1}}{P_t}+G_t+T_t=\tau_t\frac{w_t}{P_t}l_t+\frac{M_t-M_{t-1}}{P_t}+\frac{B_t}{P_t} \tag{17}$$

3 The competitive equilibrium.

The competitive equilibrium is a set of plans $\{C_t, l_t, \lambda_t, mc_t, \pi_t, v_t\}_{t=0}^{+\infty}$ that, given the policies $\{R_t, \tau_t\}_{t=0}^{+\infty}$, the exogenous processes $\{z_t, g_t\}_{t=0}^{+\infty}$, and the initial conditions, satisfies (6), (7), (8), (13), (16), (17) and the aggregate resource constraint

$$Y_t=C_t\left(1+S_t\right)+G_t+\frac{\xi_p}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\delta}}-1\right)^2+\frac{\xi_w}{2}\left(\frac{w_t}{w_{t-1}\pi_{t-1}^{\delta_w}}-1\right)^2 \tag{18}$$

4 Ramsey policy.

The Ramsey policy is a set of plans $\{R_t, \tau_t\}_{t=0}^{+\infty}$ that maximizes the expected value of (1) subject to the competitive equilibrium conditions (6), (7), (8), (13), (16), (17), (18) and the exogenous stochastic process driving the fiscal and technology shocks. Solution requires numerical simulations.[5]

4.1 The role of public expenditure variables

The first step in our analysis is to replicate the simulation exercise in SGU (2004a) with the addition that $0<\frac{T}{Y}<20\%$. Therefore, in this calibration the labour market is perfectly competitive, $\mu^w=1$, the nominal wage is flexible, $\xi_w=0$, and there is no indexation $\delta=\delta_w=0$. The time unit is meant to be a year; we set the subjective discount rate β to 0.96 to be consistent with a steady-state real rate of return of 4 percent per year; transaction cost parameters A and B are set at 0.011 and 0.075; we assume the debt-to-GDP ratio is 0.44 percent; in the goods market monopolistic competition implies a gross markup of 1.2; and the annualized Rotemberg price adjustment cost is 4.375. The preference parameter η is set so that in the flexible-price steady-state households allocate 20 percent of their time to work.

Table 2 – Baseline calibration[6]

β	$=0.96$	μ^p	$=1.20$	μ^w	$=1.00$
A	$=0.011$	ξ_p	$=4.37$	ξ_w	$=0.00$
B	$=0.075$	δ_p	$=0.00$	δ_w	$=0.00$

[4] Following SGU (2004a), $\ln g_t$, $g_t=G_t/P_t$, is assumed to evolve exogenously following an independent $AR(1)$ process. We assume that the level of the real transfer is instead exogenously given and non stochastic.

[5] These are obtained implementing SGU (2004b) 2nd order appoximation routines.

[6] In all the paper the $AR(1)$ processes driving the government spending and the technology shock are calibrated as in SGU (2004a), The serial correlation of $\ln g_t$ is set at 0.9 and the

In Figure 1 we describe the optimal inflation response to the transfer increase and to a corresponding variation in public consumption. Simulations show that inflation rapidly increases when T/Y grows beyond the 8% threshold. For instance, the optimal inflation rate is close to 3% when T/Y is 10%, and exceeds 13% when the transfer ratio is 20%. Simulations also show that in the case where public expenditure is confined to public consumption, optimal inflation would exceed 0.5% only for ratio G/Y larger than 35%.

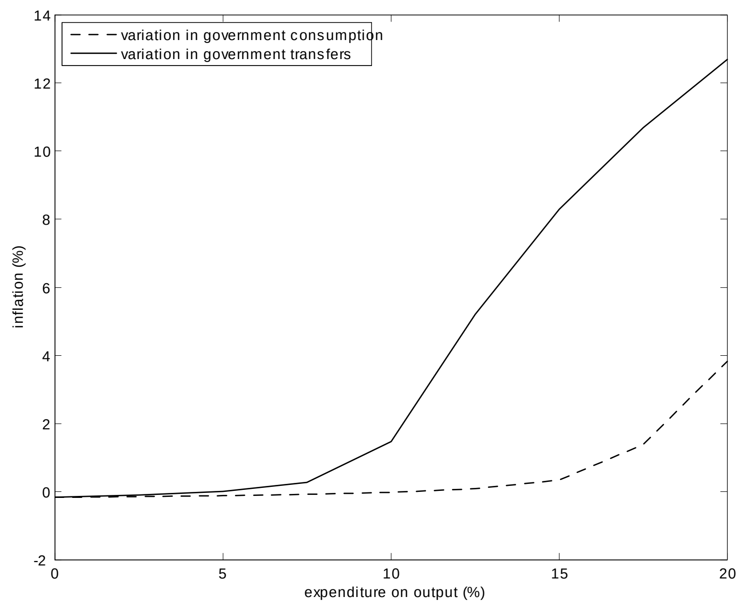

Figure 1 – Public expenditure and optimal inflation

One key mechanism driving the choice of the optimal policy mix is related to the distortionary taxation necessary to finance the additional transfers, which adversely affects the labour supply and reduces the tax base. By contrast, the increase in public consumption generates a negative wealth effect that triggers a positive labour supply response and expands the tax base. In this case the incentive to increase inflation is much reduced.

Formally, the optimal policy mix is determined by the different effects of π_t, τ_t on the policy wedge Ω_t in (14). It would be straightforward to show that $\Omega_t^{'}(\tau_t)$, $\Omega_t^{'}(\pi_t) > 0$ but $\Omega_t^{''}(\tau_t) > 0$, $\Omega_t^{''}(\pi_t) = 0$. This explains why the Ramsey planner increasingly relies on the inflation tax as public expenditures grow. In Figure 2 we compare the optimal steady state value of Ω with the

standard deviation of innovation to $\ln g_t$ is 0.0302; the serial correlation of $\ln z_t$ is 0.82 and the standard deviation of innovation is 0.0229.

value that would obtain if inflation were constrained at zero.

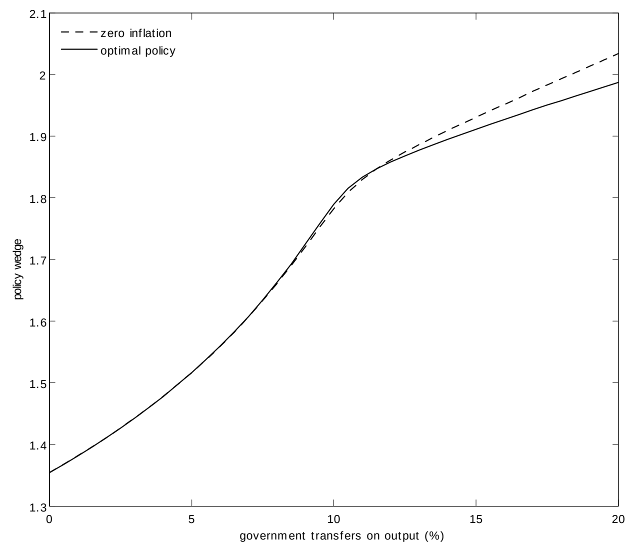

Figure 2 – Public transfers and the policy wedge

Recent studies suggest that firms adjust prices more frequently than previously thought. For instance Eichenbaum and Fischer (2007) infer that firms re-optimize prices once every 2.3–3 quarters, but cannot reject the hypothesis that firms reoptimize prices once every two quarters. In the figure below we consider the effects of different degrees of stickiness (measure as average duration of price-setting decisions) assuming that $T/Y = 10\%$. The optimal inflation rate depends on the firms' average adjustment to rest price, and substantially increase when average duration is between 2 and 3 quarters.

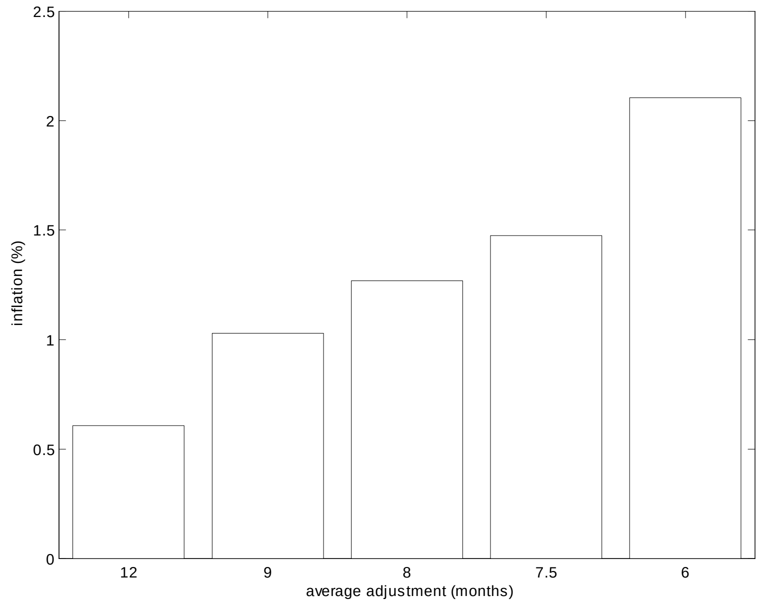

Figure 3 – Price adjustment and trend inflation

Finally, the optimal policy mix depends on monopolistic distortions. For instance, when $\mu^p = 1.1$ optimal inflation remains very close to zero for T/Y

$\leq 15\%$ (Figure 4).

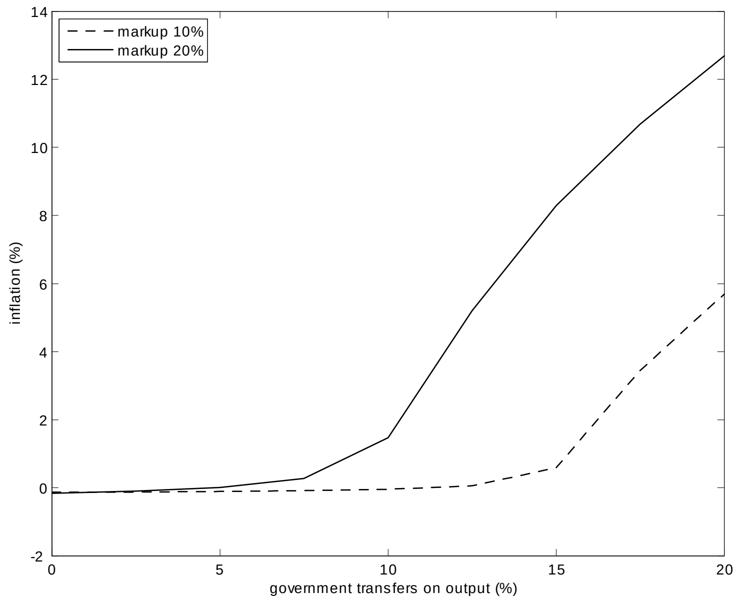

Figure 4 – Public transfers, market distortions and optimal inflation

4.2 Wage stickiness.

Introducing wage stickiness has two opposite effects on the optimal inflation rate. On the one hand, monopolistic distortions raise the incentive to substitute labor taxation with the inflation tax. On the other hand, nominal wage adjustment costs strengthen the case for price stability. After setting $\mu^w = 1.2$,[7] we postulate that price and wage adjustment costs are identical ($\xi_w = \xi_p = 4.37$). Simulations show that for $T/Y < 10\%$ the two effects offset each other (Figure 5). Beyond that threshold the wage adjustment cost dominates and the optimal

[7] Our choice of the wage markup follows Erceg *et al.* (2006), and is close to the value reported in Galí *et al.* (2007), but is lower than the calibration in Erceg *et al.* (2000). It should be noted, however, that Christiano *et al.* (2005, 2010) choose values much closer to one. We will consider a different calibration later.

inflation rate falls relative to the perfect competition case.

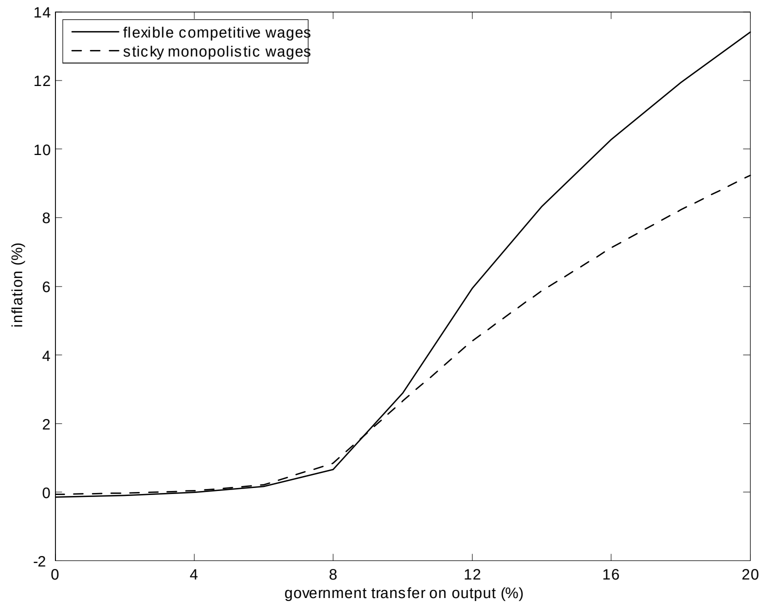

Figure 5 – Optimal inflation: Flexible vs. sticky wages

4.3 Indexation

Inflation costs associated with nominal rigidities depend crucially on assumptions about the prices set by firms that cannot reoptimize. A commonly studied indexation scheme is one whereby non-reoptimized prices increase mechanically at a rate proportional to the economy-wide lagged rate of inflation (Christiano *et al.*, 2005). In many estimated DSGE models it is assumed that the price and wage are indexed to a weighted average of past and trend inflation, in order to obtain a vertical long-run Phillips curve (see for instance Smets and Wouters, 2005, 2007). Recent contributions provide conflicting evidence on the extent of price indexation.[8] In Figure 6 we assume an identical degree of wage and price

[8] Coghley and Sbordone (2008) estimate a New Keynesian Phillips Curve, finding that price indexation in the U.S. is zero once a time-varying inflation trend is accounted for. By contrast, Barnes *et al.* (2009) show that this result is not robust to the introduction of more flexible indexation schemes. Aruoba and Schorfheide (2009) find that 15% of firms optimize in each period, 60% of firms fully index their price to past inflation, the remaining firms hold their price constant. Microdata analyses suggest that indexation parameters are lower for consumption prices than for nominal wages (Du Caju *et al.* 2008; Maćkowiak and Smets, 2008). In line with this result, Fernandez-Villaverde and Rubio-Ramirez (2008) find that $\delta = 0.15$, $\delta_w = 0.85$.

indexation ($\delta_p = \delta_w$) ranging between 0 and 40%.[9] When $T/Y > 10\%$ even a moderate degree of indexation (20%) has a non negligible impact on optimal inflation.

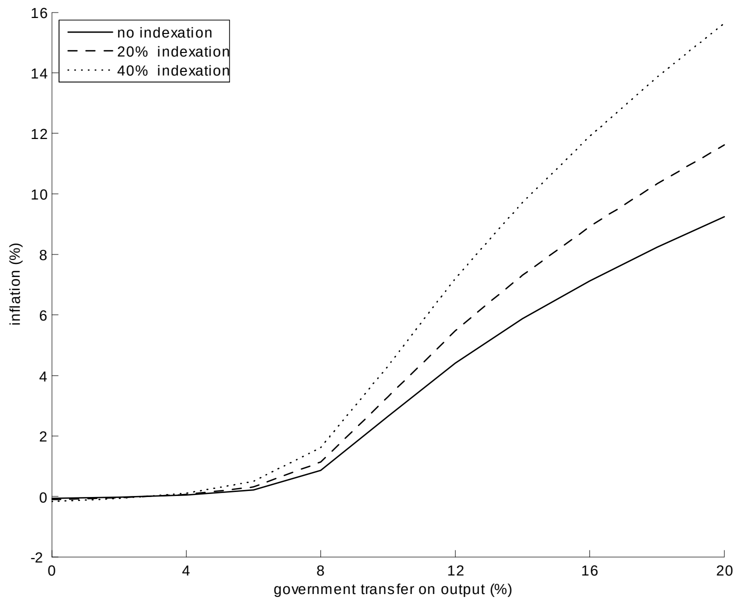

Figure 6 – Public transfers, indexation and optimal inflation

5 Extensions: Consumption scale effects in the monetary transactions technology.

The transaction cost specification adopted in (3) constrains the consumption elasticity of money demand to be one, in contrast with a large body of empirical literature. [10] Theoretical models accounting for consumption scale effects include Baumol (1952) and Khan *et al.* (2003). Attanasio *et al.* (2002) find substantial economies of scale in cash management using microdata. In a different model, Guidotti and Vegh (1993) show that the constant elasticity of scale is an unduly restrictive assumption and that it is optimal to resort to the inflation tax if the transactions costs technology does not exhibit constant returns to scale. We therefore propose a definition of $S_{t,i}$ which accounts for such scale effects.

$$S_{t,i} = s(v_{t,i})g(C_{t,i})\,; \qquad g(C_{t,i}) > 0,\ g'(C_{t,i}) < 0 \tag{19}$$

[9] Introducing asymmetries in the degrees of price and wage indexation would not affect our conclusions (simulations results available upon request).

[10] See Choi and Oh (2003), Dib (2004), Knell and Stix (2005) and references therein. Christiano *et al.* (2005) obtain an estimate of 0.1.

where $S_{t,i}$ still vanishes at v^* and $g'(C_{t,i}) < 0$[11] allows to obtain that unit transaction costs are decreasing in consumption. We assume the following specification for the monetary transaction cost[12]

$$g(C_{t,i}) = C_{t,i}^{-\theta} \quad \theta \geq 0 \tag{20}$$

Note that for $\theta = 0$ scale effects in consumption expenditure vanish and (19) converges to (4)

The resulting money demand function

$$\frac{M_t}{P_t} = \frac{C_t}{\sqrt{\frac{B}{A} + (R_t - 1)\frac{C_t^{\theta}}{A}}} \tag{21}$$

is characterized by a consumption elasticity (η_m):

$$\eta_m = \frac{\partial (M_t/P_t)}{\partial C}\frac{C}{M_t/P_t} = \left[1 - \frac{1}{2}\frac{\theta (R-1) C^{\theta}}{B + (R-1) C^{\theta}}\right] \leq 1 \tag{22}$$

This apparently innocuous modification can have substantial implications for our model. In fact condition (6) now becomes

$$\lambda_t = \frac{u_c (C_t, l_t)}{1 + S_t + C_t \frac{\partial S_t}{\partial C_t}} = \frac{u_c (C_t, l_t)}{1 + \frac{s'(v_t)v_t + (1-\theta)s(v_t)}{C^{\theta}}} \tag{23}$$

The transactions-induced wedge between the marginal utility of consumption and the marginal utility of wealth unambiguously falls in θ for any level of money velocity. Our conjecture is that this should support an increase in the optimal inflation rate.

We compare three different scenarios. In scenario 1 we represent an economy calibrated as in SGU (2004a), where parameters are calibrated as in table 2 with $G/Y = 0.2$, $T/Y = 0$. In scenario 2 instead we assume sticky wages (with $\mu^p = 1.2$ and $\xi_w = 4.37$), 20% indexation on both prices and wages, public consumption set at 20% and a transfer equal to 11% of output. In scenario 3 we assume that prices are relatively flexible and the degree of price indexation to past inflation is modest, whereas wages are by strong indexation, as found in Galí and Rabanal (2005), Rabanal and Rubio-Ramírez (2005), Fernandez-Villaverde and Rubio-Ramirez (2008) and Christiano *et al.* (2010). Relative to scenario 2, we set $\xi_p = 2.5$ (i.e., price are reset about every six months on average), $\delta_p = 0.15$ and $\delta_w = 0.85$.[13]

[11] We also assume that $g(C)$ is twice continuously differentiable.

[12] When $\theta = 0$ scale effects in consumption expenditure vanish and (19) converges to the transaction technology specified in SGU (2004a).

[13] Indexation parameters are taken from Fernandez-Villaverde and Rubio-Ramirez (2008). See below.

Table 3 – Consumption scale effects

	scenario 1		scenario 2		scenario 3	
θ	π	η	π	η_m	π	η_m
0.0	-0.15	1.000	4.43	1.000	7.87	1.000
0.4	0.00	0.959	4.63	0.962	8.26	0.962
0.8	0.12	0.956	4.80	0.963	8.55	0.963
1.2	0.19	0.967	4.92	0.974	8.95	0.974
1.6	0.23	0.978	4.98	0.984	9.13	0.984
2.0	0.25	0.987	5.00	0.991	9.22	0.991

Our simulations (Table 3) confirm that optimal trend inflation is increasing in θ. The strongest impact on inflation is obtained in scenario 3, when price and nominal wage adjustment costs are relatively milder. In steady state equilibrium consumption scale effects have a limited, reversed hump-shaped effect on consumption money demand elasticity, which reaches a minimum value for about $\theta = 0.6$.

6 A calibration for the US economy.

In this section we calibrate the model to the US economy. Our purpose is to benchmark the optimal inflation rate against Fernandez-Villaverde and Rubio-Ramirez (2008) estimates the time-varying inflation target implicitly adopted by the Federal Reserve over the period 1957-2000 and over the high inflation sub-sample 1973-1991. [14] The ratios G/Y and T/Y are derived from the US NIPA dataset. During the period 1957-2000 the average government-consumption- and transfers-to-GDP have respectively been 20% and 9%. For the sub-sample 1973-1991 we find similar figures for G/Y and a slightly higher transfers ratio, about 10%. [15] As before we assume that the subjective discount rate β is 0.96 and the transaction cost parameters A and B are 0.011 and 0.075. For the remaining parameters $\left(\theta, \xi_p, \xi_w, \delta_p, \delta_w, \mu^p, \mu^w\right)$ we consider 6 alternatives (Table 4) . The first calibration simply replicates the SGU 2004a exercise. Thus we have perfect competition in the labor market and no indexation. The second calibration differs from the first because we consider consumption scale effects in monetary transaction costs to the calibration. The third calibration extends the second one by introducing in the labor market monopolistic competition and nominal rigidities which are identical to those assumed for the goods market. In addition, we allow for a moderate degree of price and wage indexation (25%). In calibration 4 the parameters describing nominal rigidities $\left(\xi_p, \xi_w\right)$ imply that prices re-optimized on average every 10 months and wages every 9 months as in Smets and Wouters (2007). In calibration 5 we consider the highest frequency of price adjustment we found in the literature, 2 quarters, as reported in and

[14] On the relevance of inflation time-varying targets for monetary policy see Taylor (1998), Sargent (1999), Primiceri (2006), Cogley and Sbordone (2008).

[15] As shown above, beyond the 8% threshold even a modest increase in $\frac{T}{Y}$ may have a strong impact the optimal inflation rate.

Eichenbaum and Fisher (2007).[16]

Table 4 – The US economy calibration

Fixed parameters		Alternative calibrations					
			(1)	(2)	(3)	(4)	(5)
β	$= 0.96$	θ	0	2	2	2	2
A	$= 0.011$	ξ_p	4.37	4.37	4.37	7	2.47
B	$= 0.075$	ξ_w	0	0	4.37	9.5	4.37
		δ_p	0	0	0.25	0.25	0.25
		δ_w	0	0	0.25	0.25	0.25
		μ^p	1.2	1.2	1.2	1.2	1.2
		μ^w	1	1	1.2	1.2	1.2

Simulations show that for all calibrations the optimal inflation rate is positive and increasing in the sub-sample 1973-1991 (Table 5). In this regard, it is interesting to note that the optimal inflation rate is highly sensitive to the small change in T/Y observed over the two samples. A comparison between calibrations 1 and 2 highlights the role of consumption scale effects in monetary transaction costs. Differences in the price optimization inertia obviously explain differences in the optimal inflation rate. Simulation 3 and 5 seem to provide the best approximations to the estimated targets.

Table 5 – Optimal, observed and targeted inflation[17]

	US economy		scenario				
	observed*	est. target	(1)	(2)	(3)	(4)	(5)
(1) whole sample	4.4	3.2	1.4	2.7	3.4	2.0	4.0
(2) high inf. period (73-91)	6.4	5.6	2.9	3.9	4.3	2.7	5.2
(*) CPI inflation, excluding food and energy.							

7 Conclusions.

Since Phelps we know that a positive inflation rate might mitigate the distortions induced by need to finance government budgets. In contrast with previous research, we show that this argument is relevant given the policy mix between government consumption and transfers that we observe in OECD countries.

[16] In calibrations 4 and 5 we maintain a 25% degree of price and wage indexation because both Smets and Wouters (2007) and Eichenbaum and Fisher (2007) assume full indexation in steady state, thus obtaining a long run vertical Phillips curve. Fernandez-Villaverde and Rubio-Ramirez (2008) obtain estimates for $\xi_p, \xi_w, \delta_p, \delta_w$ starting from flat priors. We do not consider here their reported values because the variant of calvo pricing they consider imposes a constant elasticity of substitution across goods over the business cycle and overestimates the degree of price inertia. For a criticism of their appproach see Limball (1995) and Eichembaum and Evans (2007).

[17] The estimated targets are computed from Fernandez-Villaverde and Rubio-Ramirez (2008). They report the targets for the whole period 3.2% and discuss that the target was 1.6% in the period between 1950-72 and in the 90'. From these information one can derive the target for the high-inflation period (1973-91). See also figures 2.4 and 2005 in their paper.

This result holds for plausible parameterization of price and nominal wage adjustment costs. The size of monopolistic distortions, the degree of price and wage indexation, the consumption scale effect in monetary transaction costs unambiguously increase the optimal inflation rate. Unfortunately, empirical evidence on these latter variables is rather limited. In fact estimated DSGE models typically impose markup parameters, assume a vertical long-run Phillips curve and neglect monetary transaction costs.

Calibrations show that the prediction of a positive inflation rate holds for the US, where government size is relatively small. *A fortiori*, our reconsideration of the Phelps conjecture appears even more appropriate when considering countries in the Euro area where the welfare state plays a more important role. In contrast with SGU (2010), who argue that central bank inflation targets are too high, our contribution shows that a 2% target might be too low, at least for countries where the burden of taxation is rather high, such as continental Europe. The explanation for this might be that commitment to a low inflation rate is used to discipline spending decisions, assumed exogenous in our model. In fact several political economy models point out that distorted policymakers' incentives inflate public expenditures.[18] As shown in Acemoglu *et al.* (2009), the Ramsey-optimal taxation is substantially affected when taxes and public good provision are decided by a self-interested politician who cannot commit to policies. In a similar vein, further research should investigate how these two frictions, i.e. politicians' self-interest and lack of commitment, may affect the choice of the optimal inflation target.

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[18] See Tornell and Lane (1999) and Persson and Tabellini (2003, 2004).

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