A generalized no-arbitrage principle in a finite setting

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Introduction

The classical one-period no-arbitrage pricing theory assumes that:

- the market is frictionless and competitive,
- there exists a reference probability P.

The classical **no-arbitrage condition** is equivalent to the existence of a **linear pricing rule**, that can be expressed as a **discounted expectation** with respect to an equivalent risk-neutral probability $Q \sim P$.

Goal

We look for a pricing rule that allows to model **frictions** in the financial market: such pricing rule has to be, generally, **non-linear**.
Hence, dropping the fundamental assumptions of the classical no-arbitrage theory, a generalization of the no-arbitrage principle is required.

Frictions

Real markets show the presence of frictions such as transaction costs, taxes, bid-ask spreads. Generally, in frictional market models the pricing rule is non-linear.

Chateauneuf et al. (1996) firstly consider a **lower (upper)** pricing rule π that can be expressed as the discounted Choquet integral with respect to a **convex (concave)** non-additive risk-neutral capacity ν:

$$\pi(\cdot) = (1+r)^{-1}\mathbb{C}_\nu(\cdot). \tag{1}$$

For a convex ν, the Choquet expectation can be interpreted as a lower expectation computed among probability measures in the core of ν

$$\mathbb{C}_\nu(\cdot) = \min_{P \in \mathrm{core}(\nu)} \mathbb{E}_P(\cdot), \text{ where } \mathrm{core}(\nu) = \{P \text{ probability } : P \geq \nu\}.$$

Choquet pricing rules, generally, do not guarantee that the market is free of arbitrage in the classical sense.

Generalized no-arbitrage (1)

Given a functional $\pi : \mathbb{R}^\Omega \to \mathbb{R}$, a series of papers aimed at determining when π can be expressed as the (discounted) Choquet integral with respect to a convex (concave) capacity.

Chateauneuf et al. (1996) and Cerraia-Vioglio et al. (2015) consider two forms of put-call parity in relation to Choquet pricing:

$$\pi(X_T) = \pi(C_T) + \pi(-P_T) + \pi(k1_\Omega). \tag{CV}$$

$$\pi(P_T) = \pi(C_T) + \pi(-X_T) + \pi(k1_\Omega), \tag{CH}$$

(CV) characterizes a Choquet pricing rule, while (CH) is equivalent to (CV) and no bid-ask spreads.

Bastianello et al. (2022) and Chateauneuf and Cornet (2022) prove that a Choquet pricing rule satisfies a form of no-arbitrage if and only if there exists a probability dominated by the capacity associated to the pricing functional.

Generalized no-arbitrage (2)

Previous approaches consider a pricing rule already defined on the set of all random variables $\mathbb{R}^\Omega$. The most common situations is to have a lower (upper) price assessment only on a finite set of reference assets.

Cinfrignini et al. (2021) obtained a generalization of the **first fundamental theorem of asset pricing** in a one-period belief function space, proving that generalized no-arbitrage is equivalent to existence of an equivalent risk-neutral belief function whose discounted Choquet expectation agrees with the given lower prices.

Partially resolving uncertainty (PRU) assumption Jaffray (1989)

The portfolio's payoff is defined assuming that the agent may acquire the information that an event $B \neq \emptyset$ occurs without knowing which is "true" state $\omega \in B$.

Conclusions and future perspectives

- The generalized no-arbitrage condition is equivalent to the existence of a (positive) belief function whose discounted Choquet expectation agrees with the given lower prices.
- Our aim is to improve the pricing rule in terms of discounted Choquet expectation extending the notion of arbitrage opportunity and theorems connected with it from the one-period to the multi-period setting.
- We need to define a Choquet expectation $\mathbb{C}[\cdot|\mathcal{F}_t]$ conditional to a filtration $\{\mathcal{F}_t\}$ satisfying suitable versions of Markov, time-homogeneity and martingale properties.

Thank you for the attention!

Partial references (1/2)

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