Stochastic analysis of a government issue in currency

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Outline

Stochastic analysis of a government bond in currency

Introduction

On April 27, 2021, the Treasury Department of the Italian Ministry of Economy and Finance announced the issue for 1.5 billion US dollars of the thirty-year Italy Govt bond with ISIN US465410CC03, maturing May 6 2051 and coupon rate of 3.875 % paid every six months. The issue fix the price of 98.89 USD to which corresponded a yield of 3.94%. The issue raised 6.2 billion US dollars.

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Relative analysis

Respect to:

- Btp Tf 1,7% Sep 1 51 Eur with coupon rate of 1,7% we have a spread about 181 bps.
- Treasury thirty-year we have a spread about 157 bps.

These spreads suggest us to investigate the risk-return profile of this security.

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The view of investment

The assumption is that this security can be purchased by a Pension Fund with a cash flow matching view and therefore held to maturity, i.e. the investor doesn't care about the price change that occurs until the maturity.

Mapping of the risk

The risk is linked to an increase in the domestic currency respect to the foreign currency which would decrease the internal rate of return due to the reduction of the amounts associated with the cash flow.

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The Cross-Rate Eur/Dollar

It's linked to:

- inflation rate
- interest rate
- current account balance (surplus/deficit)

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Through the law of Purchasing Power Parity we can check if the current level is aligned with the equilibrium level or not.

Considering both **"mature"** economies, we can assume that any anomalous oscillations, both in size and in speed, are monitored by the respective Central Banks in order to avoid shocks on the real economy.

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The data

From Bloomberg we obtain the USD/EUR exchange rate time series, on a monthly frequency, from 31/01/1975 to 30/04/2021. The time series is composed by 556 observations.

After a previous statistical/econometric analysis, different stochastic processes are presented and calibrated by **maximization of the likelihood function** to simulate the cross-rate dynamic.

Quantitative analysis of the currency risk associated with the investment

The process and parameters

- Geometric Brownian Motion $dx_t = \mu x_t dt + \sigma x_t dw_t$

$\hat{\mu}$	$\hat{\sigma}$
0.0077	0.1021

- Variance-Gamma $dlogx_t = \mu dt + \theta dg_t + \sigma dw(g_t)$

$\hat{\mu}$	$\hat{\sigma}$	$\hat{\nu}$	$\hat{\theta}$
-0.006	0.0295	0.32	0.0062

- Vasicek $dx_t = \alpha(\theta - x_t)dt + \sigma dw_t$

$\hat{\alpha}$	$\hat{\theta}$	$\hat{\sigma}$
0.1947	0.8745	0.09

- Exponential Vasicek $dx_t = \alpha x_t(m - log(x_t))dt + \sigma x_t dw_t$ with $m = \theta + \frac{\sigma^2}{2\alpha}$

$\hat{\alpha}$	$\hat{\theta}$	$\hat{\sigma}$
0.219	0.1473	0.1017

Quantitative analysis of the currency risk associated with the investment

Currency risk valuation

For each coupon payment date and for redemption , the corresponding security dollar cash flow is converted based on the USD / EUR currency rate value for each simulated scenario.

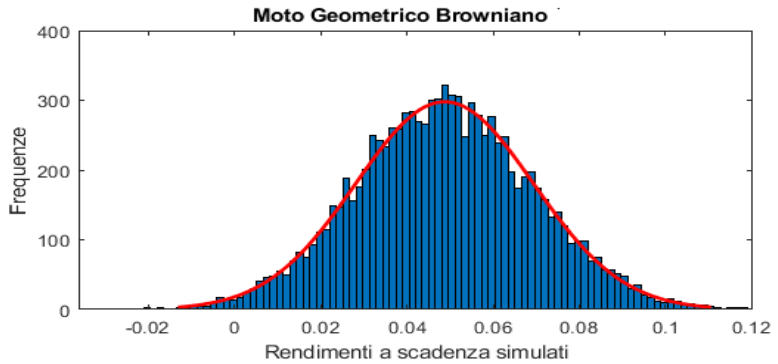
Output

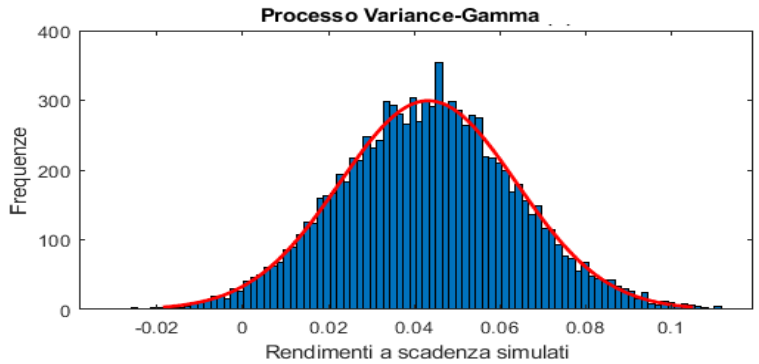
- the probabilities that the security will have a negative yield to maturity;
- the probabilities that the security will have a yield to maturity lower than the corresponding yield to maturity of the bond denominated in euro object of relative analysis;
- the percentiles of simulated yields to maturity.

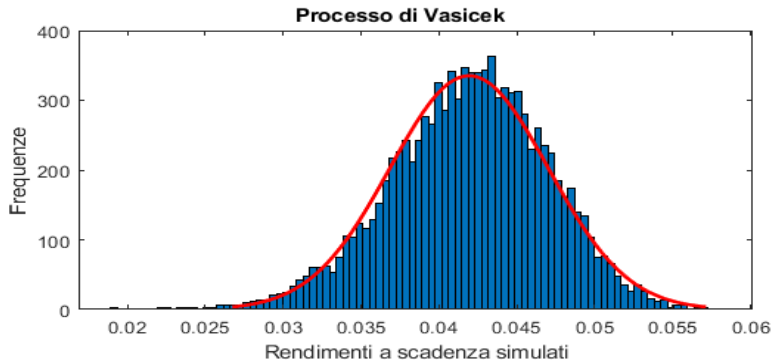
Quantitative analysis of the currency risk associated with the investment

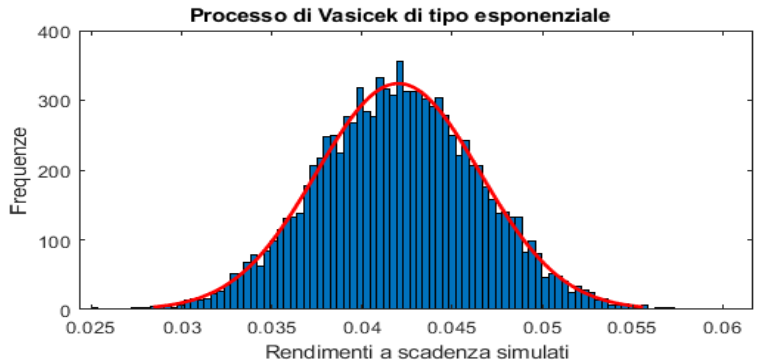
Empirical evidences

Distributions of yields to maturity obtained by simulating 10,000 trajectories of the monthly USD / EUR exchange rate for each of the stochastic processes considered.









Results and Conclusions

Shortfall Probabilities

	Shortfall Probabilities	
	$\text{YTM}_{sim} < 0$	$\text{YTM}_{sim} < YTM_{BTP\,1.7}$
GBM	0,84%	8,04%
Variance-Gamma	1,60%	13,09%
Vasicek	-	0,03%
Exp Vasicek	-	-

Percentiles

	Percentiles simulated						
	1	5	10	50	90	95	99
GBM	0,17%	1,51%	2,26%	4,86%	7,55%	8,32%	9,68%
Variance-Gamma	-0,39%	0,96%	1,69%	4,33%	6,96%	7,78%	9,26%
Vasicek	2,93%	3,32%	3,53%	4,22%	4,83%	4,98%	5,28%
Exp Vasicek	3,17%	3,45%	3,62%	4,20%	4,79%	4,94%	5,26%
$\text{YTM}_{BTP\,1.7}$	1,95%						

Results and Conclusions

Main conclusions

- From the simulations, taking volatility into account, we have that the spread between the security denominated in currency and the security denominated in euro can justify the investment.
- Recalling the economic assumptions we have that the both Vasicek and Exponential Vasicek models, through the mean reverting, can better reflect the exchange rate behavior.

Reference

- C. Di Palo, P. Fava, *ANALISI STOCASTICA DI UNA EMISSIONE IN VALUTA*, accepted by Annali-Memotef 2022.