Optimal annuitization and bequest motives

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The Annuity Puzzle

Theoretical fact: (Yaari 1965) showed that individuals with no bequest motive would convert their wealth entirely into an annuity.

Empirical fact: There is a low demand for annuities and the annuity market is thin (Peijnenburg 2016), (Boyer 2020).

Possible explanations: The adverse selection, the annuity costs, the irreversibility features of the life annuity investments, the wealth level, and incomplete annuity markets.

Our research would investigate the impact of bequest motives on annuitization decisions.

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The model: Financial aspects

We consider an individual whose retirement wealth is invested in a financial fund that can be converted into an annuity.

The value $(X_t)_{t\geq 0}$ of the financial fund evolves according to:

$$\begin{cases} dX_t = (\theta - \alpha)X_t dt + \sigma X_t dB_t \\ X_0 = x > 0 \end{cases}$$

where

- θ is the average continuous return of the financial investment,
- α is the constant dividend rate,
- σ is the volatility coefficient,
- B_t is a standard Brownian motion.

The model: Actuarial aspects (1)

Let τ_d be the time of the individual's death.

The probability that a person at time t survives the next z years is:

$$_zp_t := \mathbb{P}(\tau_d > t + z | \tau_d > t) = \exp\left(-\int_t^{t+z} \mu_s ds\right)$$

μ_s is called *subjective* mortality force and it could be:

- **Constant:** The probability of surviving z years is independent of the individual's age.
- **A deterministic function:** It takes into account the passage of years and the probability of death increases with age.
- **A stochastic process:** It takes into account the unpredictability.

The model: Actuarial aspects (2)

The insurance company uses an *objective* mortality rate $\hat{\mu}$ (which is public information provided by a demographic analysis of the population) to price the annuity.

An annuity that pays at a rate of one monetary unit per year is:

$\hat{a}_t := \int_0^\infty e^{-\hat{\rho}s} {}_s\hat{p}_t ds$

If the individual decides to annuitize, his **entire** income X_t will be converted into an annuity. The constant payment deriving from an annuity purchase would be

$$P_t := \frac{X_t - K}{\hat{a}_t}$$

K is a fixed acquisition fee ($K > 0$) or a tax incentive ($K < 0$).

The agent's optimization problem

Let τ is the **time of the annuity purchase**.

- In $[0, \tau_d \wedge \tau]$ the individual receives the dividends from the fund at rate α.
- In $[\tau_d \wedge \tau, \tau_d]$ he gets the annuity payment at a constant rate P_τ per year.
- On $\{\tau_d \leq \tau\}$, she leaves a bequest equal to her wealth.

The value function of the optimization is defined by:

$$V(x) = \sup_{\tau} \mathbb{E}\left[\int_0^{\tau_d \wedge \tau} e^{-\rho t} \alpha X_t dt + \mathbb{1}_{\{\tau_d \leq \tau\}} e^{-\rho \tau_d} \nu X_{\tau_d} + P_\tau \int_{\tau_d \wedge \tau}^{\tau_d} e^{-\rho t} dt\right]$$

- ρ is the individual's constant discount rate for the future cash flows,
- $\nu \in [0, 1]$ measures the **strength of the bequest motives**.

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The optimal stopping problem

Assumption

- Financial and demographic risks are independent.
- Both the individual and the insurer uses **constant** mortality forces throughout the lifetime.

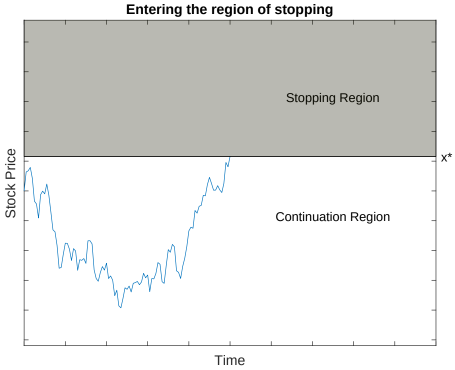
Mathematically the problem is formulated as an optimal stopping problem.

We can define:

- **the continuation region** C**:** It is convenient for the individual to invest in the financial market and not buy the annuity.
- **the stopping region** S**:** It is convenient for the individual to buy the annuity.

When annuitize?

a If $K \geq 0$ and $\nu < \nu^*$ then then there exists x^* such that $\mathcal{C} = (0, x^*)$ and $\mathcal{S} = [x^*, \infty)$.

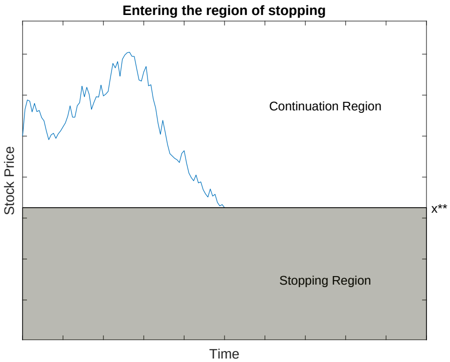


b If $K \geq 0$ and $\nu \geq \nu^*$ then $\mathcal{C} = (0, \infty)$, and it is never optimal to purchase an annuity.

When annuitize?

c If $K < 0$ and $\nu < \nu^*$ then $\mathcal{S} = [0, \infty)$, i.e. the annuity is immediately purchased whatever is the initial wealth x.

d If $K < 0$ and $\nu \geq \nu^*$ then there exists x^{**} such that $\mathcal{C} = (x^{**}, \infty)$ and $\mathcal{S} = [0, x^{**}]$.



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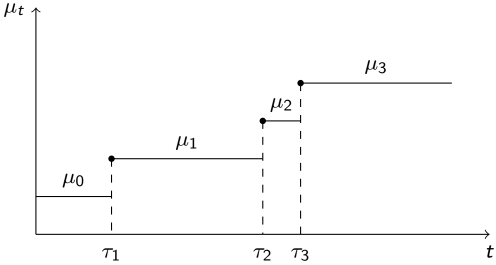
A stochastic mortality force

Let τ_i be a sequence of random variables i.i.d. with exponential distribution with parameter λ_i, we define the *subjective* mortality force function μ_t as

$$\mu_t := \mu_0 + \Delta_\mu \sum_{i \geq 1} \mathbb{1}_{\{t \geq \tau_i\}}$$



How does this impact the results obtained?

Thank you very much for your attention!