Pension fund with longevity risk: an optimal portfolio insurance approach

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Introduction and aim

- Pension funds offer **social insurance** by providing income to the fund members once retired, aiming to **maintain living standards**.
- Pension fund activity is affected by two main risks: the **investment risk**, concerning the **accumulation phase**, and the **longevity risk**, concerning the **decumulation phase**.
- Is it possible to guarantee a minimum retirement saving at the retirement date by linking the investment strategy (asset allocation during the accumulation period) to the fund member's lifetime (longevity paths during the decumulation period)?

Research goal

Design an optimal asset allocation strategy for guaranteeing a minimum retirement saving, taking into account the longevity risk.

The state variables

On a continuously open and frictionless financial market over the time set $[0, T)$, the economic framework is described by two **state variables**:

1. the **instantaneous risk-free rate** $r(t)$ whose dynamics is given by a **CIR process**:

$$dr(t) = k\,(\theta - r(t))\,dt + \sigma_r \sqrt{r(t)}dZ_r(t), \quad r(0) = r_0 > 0, \quad t \in [0, T], \tag{1}$$

2. the **mortality intensity** of the workers, which are assumed to be homogeneous by cohort evolves according to

$$d\lambda(t) = \alpha_\lambda\,(\beta_\lambda(t) - \lambda(t))\,dt + \sigma_\lambda \sqrt{\lambda(t)}dZ_\lambda(t), \quad \lambda(0) = \phi + \frac{1}{b}e^{\frac{\iota+t-l}{b}}, \quad t \in [0, T], \tag{2}$$

where

$$\beta_\lambda(t) = \phi + \frac{1}{b}\left(\frac{1}{\alpha_\lambda}\frac{1}{b} + 1\right)e^{\frac{\iota+t-l}{b}}.$$

Labour income and contribution rate processes

- We assume that the member's **labour income** $L(t)$ is stochastic and its evolution can be expressed in terms of the following SDE

$$\frac{dL(t)}{L(t)} = \zeta dt + \sigma_{L,r}\sqrt{r(t)}\left(dZ_r(t) + \xi_r\sqrt{r(t)}dt\right) + \sigma_L\left(dZ_S(t) + \xi_S dt\right), \quad t \in [0, T], \tag{3}$$
$$L(0) = l_0 > 0.$$

- The **total contribution** $C(t)$ is given by

$$C(t) = \gamma^\star L(t), \quad t \in [0, T]. \tag{4}$$

where $\gamma^\star \in (0, 1)$ is set such that the **fairness condition** is satisfied at $t = 0$

$$\mathbb{E}^{\mathbb{Q}}\left[\int_0^{\omega'} \left(C(s)\mathbb{1}_{s<T} - b(s)\mathbb{1}_{s\geq T}\right) e^{-\int_0^s r(u)du}\frac{p(s)}{p(0)}ds\right] = 0 \tag{5}$$

The financial market

In the financial market, the DC pension scheme's manager can allocate the wealth of the pension account into

- a **money market account** (cash) whose dynamics is given by

$$dS_0(t) = r(t)S_0(t)dt, \quad S_0 = s_0, \quad t \in [0, T], \tag{6}$$

- a **risky asset** whose price evolves according to the following SDE

$$\frac{dS(t)}{S(t)} = r(t)dt + \sigma_{S,r}\sqrt{r(t)}\left(dZ_r(t) + \xi_r\sqrt{r(t)}dt\right) + \sigma_S\left(dZ_S(t) + \xi_S dt\right), \quad t \in [0, T], \tag{7}$$
$$S(0) = s > 0.$$

- A **rolling ZCB** $P_K(t)$ with a constant time to maturity K, whose price dynamics is given by

$$\frac{dP_K(t)}{P_K(t)} = r(t)dt - \sigma_K\sqrt{r(t)}\left(dZ_r(t) + \xi_r\sqrt{r(t)}dt\right), \quad t \in [0, T]. \tag{8}$$

The financial market (II)

- The third risky asset in the financial market is a zero-coupon longevity bond, which is primarily used to hedge **longevity risk**.

Definition 1 (Zero coupon longevity bond)

A zero-coupon longevity bond is a contract paying a face amount equal to the survival probability of the reference population from time 0 until a fixed maturity time s. Its **arbitrage-free price** at time t for a fixed maturity s is given by

$$L_B(t,s) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)du}\frac{p(s)}{p(0)}\right] = e^{-\int_0^t \lambda(u)du}\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)+\lambda(u)du}\right], \quad 0 \leq t < s \leq T. \tag{9}$$

- In the same manner of the zero coupon bond, we consider a **rolling longevity bond** $L_{K_1}(t)$ with a constant time to maturity K_1, whose dynamics is given by

$$\frac{dL_{K_1}(t)}{L_{K_1}(t)} = r(t)dt - \sigma_{K_1}\sqrt{r(t)}\left(dZ_r(t) + \xi_r\sqrt{r(t)}dt\right) - \sigma_{L_{K_1}}\sqrt{\lambda(t)}\left(dZ_\lambda(t) + \xi_\lambda\sqrt{\lambda(t)}dt\right). \tag{10}$$

The investment strategy during the accumulation phase

- Generally, the aim of the pension fund's manager is to reach a **minimum guarantee** to purchase a **lifetime annuity** for the surviving member at retirement time T. Its value at time T is given by

$$G(T) = \mathbb{E}^{\mathbb{Q}}\left[\int_T^{\omega'} b(s) e^{-\int_T^s r(u)du} \frac{p(s)}{p(T)} ds \middle| \mathcal{F}_T\right]. \tag{11}$$

- One of the most popular strategy with downside protection is the so–called **CPPI strategy** which works as follows:
  1. **Floor**: $dF(t) = r(t)F(t)dt, \quad F(0) = \mathcal{G} \exp\left\{\int_0^T r(u)du\right\}, \quad t \in [0, T],$
  2. **Cushion**: $C(t) = W(t) - F(t), \quad t \in [0, T],$
  3. **Exposure**: $E(t) = m \cdot C(t), \quad t \in [0, T]$, where m is the **multiplier**.

- We propose a generalized version of CPPI strategy, the so-called **purpose–oriented proportional PI strategy** which presents the following features:

1. the floor is directly linked to the target annuity value at retirement T,

$$G(t) = \mathbb{E}\left[G(T)e^{-\int_t^T r(u)du} \middle| \mathcal{F}_t \right] = \int_T^{\omega'} b(s)L_B(t,s)ds, \quad t \in [0,T], \tag{12}$$

2. the stock index is replaced by a synthetic index consisting of a linear combination of three different kinds of underlying:

$$I(t) = (1 - \alpha_1(t) - \alpha_2(t))\, S(t) + \alpha_1(t)P_K(t) + \alpha_1(t)L_{K_1}(t), \quad t \in [0,T]. \tag{13}$$

- The pension fund aims to maximize the expected utility of the terminal surplus at time T:

$$\begin{cases} \text{maximize } \mathbb{E}^{t,w,r,\lambda}\left[\dfrac{(W^{\nu}(T) - G(T))^{1-\delta}}{1-\delta} \right] \text{ over } \nu = \{\alpha_1(u), \alpha_2(u), m(u)\}_{u \in [t,T]} \in \mathcal{A}(t,w,r,\lambda), \\ \text{such that } W^{\nu}(T) \geq G(T), \end{cases} \tag{14}$$

where the dynamics of the wealth process $W(t)$ is given by

$$dW(t) = W(t)\frac{dS_0(t)}{S_0(t)} + m(t)\,(W(t) - G(t))\left(\frac{dI(t)}{I(t)} - \frac{dS_0(t)}{S_0(t)} \right) + p(t)C(t)dt. \tag{15}$$

Numerical analysis

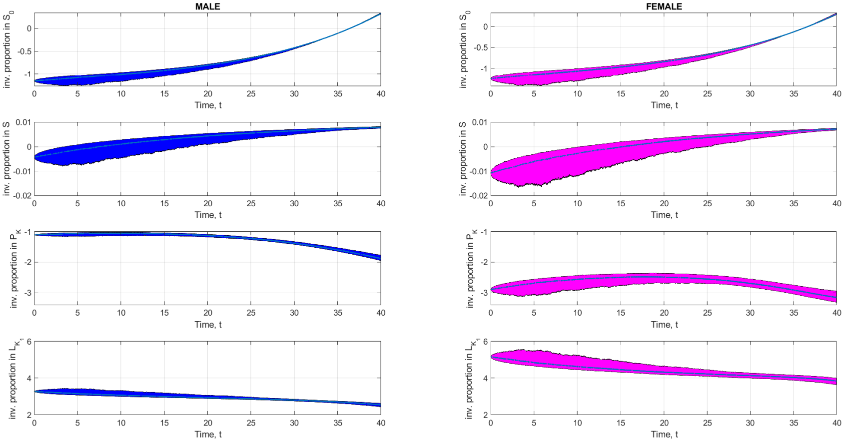

Figure: Median paths of optimal investment proportion with $\delta = 3$.

Conclusion

- We studied the optimal investment problem for a DC pension scheme in a framework where both interest rate risk and longevity risk are considered.
- Our theoretical results and subsequent numerical studies showed evidence that the longevity bond plays an important role in DC scheme's risk management.
- We observed that more risk-averse the scheme manger, lower the investment proportion in longevity bond. However, even for a highly risk-averse manager, we showed that it is optimal to invest a large proportion of the scheme's wealth in the longevity bond.

Thanks for the attention!