Mathematics and finance: how to hedge risk with portfolio insurance strategies

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Portfolio Insurance Strategies

Portfolio Insurance strategies are financial tools whose aim is to hedge against downside risk or to guarantee an equity market participation in case of favorable market conditions.

Due to the 2007/08 worldwide financial crisis, both retail and institutional investors faced a new challenge: to guarantee high participation rates, despite having very low interest rate levels.

Constant Proportion Portfolio Insurance

- Floor process: $dF_t = r_t F_t dt$, where $F_0 = G \cdot PL \cdot p(t, T, r)$.
- Cushion: $C_t = \max\left\{0; V_t^{CPPI} - F_t\right\}$.
- Exposure: $E_t = M \cdot C_t$, where $M \in \mathbb{R}^+$ is the multiplier.

The CPPI portfolio consists of a risky asset S_t and a risk-free asset B_t, such that the CPPI portfolio, V_t^{CPPI} is given by

$$V_t^{CPPI} = \alpha_t^{CPPI} S_t + (1 - \alpha_t^{CPPI}) B_t, \ \forall t \in [0, T], \tag{1}$$

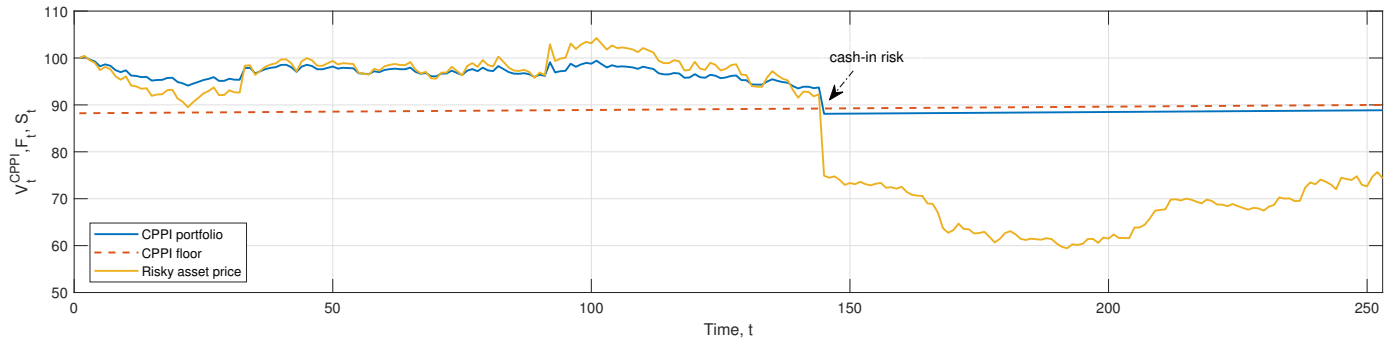
where

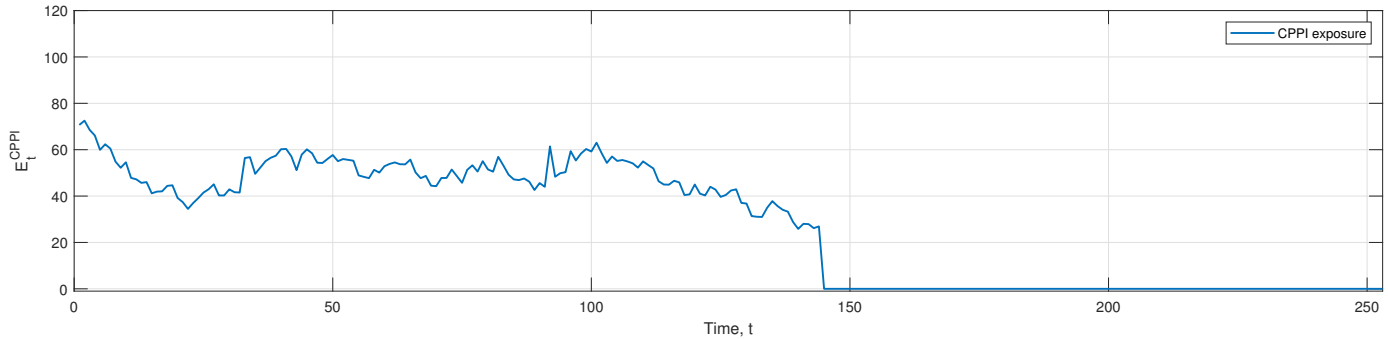
$$\alpha_t^{CPPI} = \frac{E_t}{V_t^{CPPI}} = \min\left\{L_{max}, \frac{MC_t}{V_t^{CPPI}}\right\}, \forall t \in [0, T], \tag{2}$$

where L_{max} is the so-called maximum leverage factor.

The cash-in risk

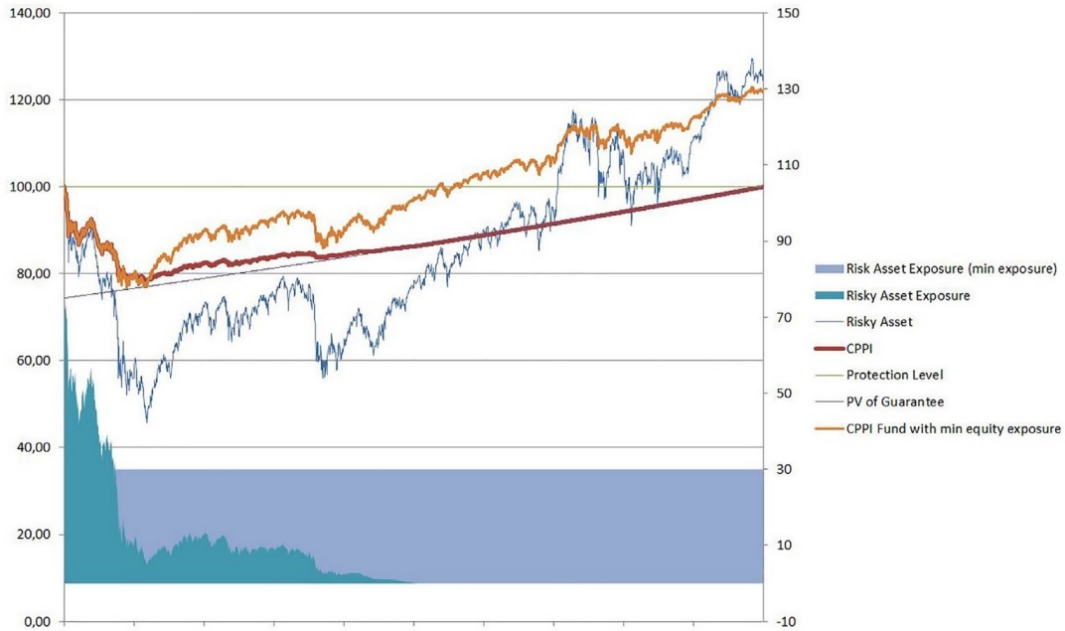
- **cash-in risk:** a non-zero probability that the portfolio value falls below the floor level failing to guarantee $G \cdot PL$.





How to annihilate such a risk?

The GMEE-CPPI strategy



The G-CPPI portfolio consists of a risky asset S and a risk-free asset B such that the proportion of wealth invested into the risky asset α_t^{G-CPPI}, is given by

$$\alpha_t^{G-CPPI} = \max\left\{\min\left\{L_{max}, \frac{MC_t}{V_t^{CPPI}}\right\}, \alpha_{min}\right\}, t \in [0, T] \qquad (3)$$

where $\alpha_{min} \in [0, 1]$ is the guaranteed minimum equity exposure.

OBPI-CPPI mixture: GOC STRATEGY

1. A significant proportion of portfolio is invested in time congruent ZCB through OBPI (downside protection).
2. The remaining part of the portfolio is put into call options written on a G-CPPI strategy (equity market participation).

The GOC portfolio is

$$V_t^{GOC} = PL \cdot G \cdot p(t, T, r) + \xi \cdot O_t(t, V_t^{G-CPPI}, K), \ t \in [0, T]. \quad (4)$$

where

$$\xi = \left(\frac{G(1 - PL \cdot p(0, T, r))}{O_0} \right)^+ . \quad (5)$$

Thank you for your attention!

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