Cryptocurrencies: trading and pricing

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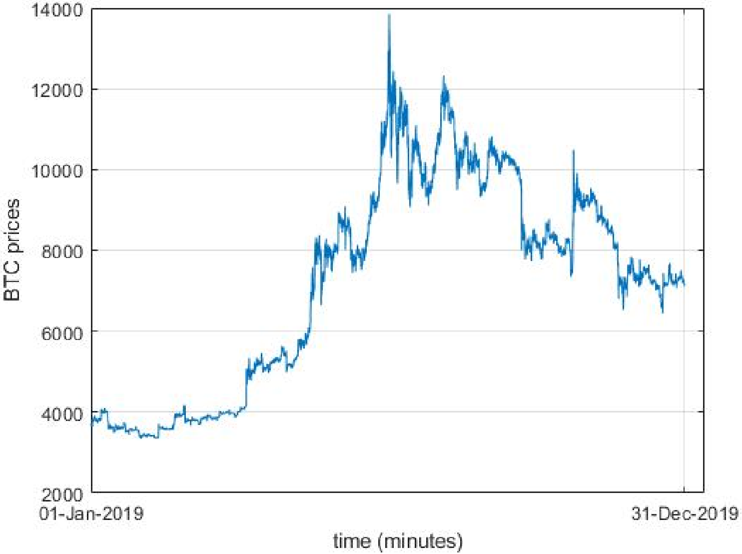
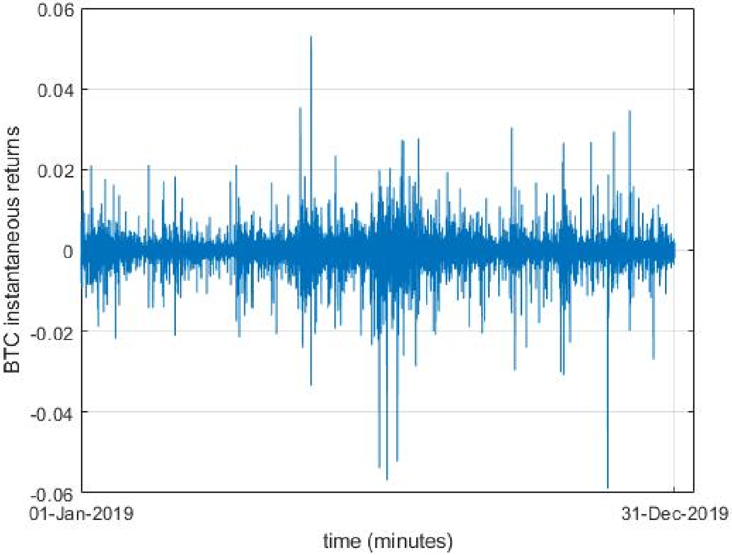
Cryptocurrencies

- In the last decade, the role of cryptocurrencies has been increasing its prominence in the worldwide economic and financial scenario.
- Since the introduction of Bitcoin (BTC) in 2009, there are over six thousand of digital currencies on the market.
- As proof of this, it is sufficient to note that the 2019 BTC market capitalization was equal to about 53 of the leading cryptocurrencies market capitalization.

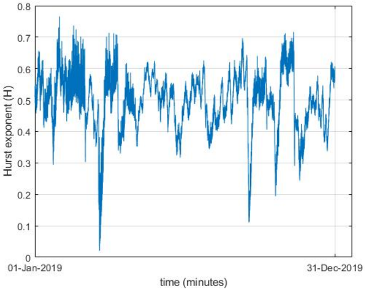
Bitcoin


14000
12000
10000
8000
6000
4000
2000
BTC prices
01-Jan-2019
31-Dec-2019
time (minutes)

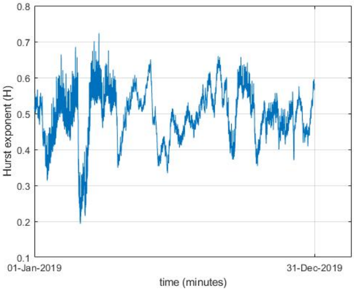
BTC instantaneous returns



Empirical findings



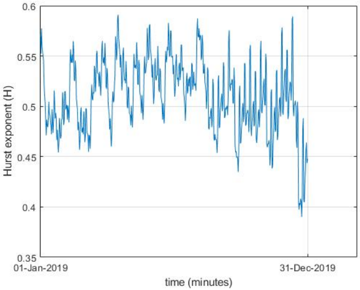
(a) Hurst exponent (1 hour)



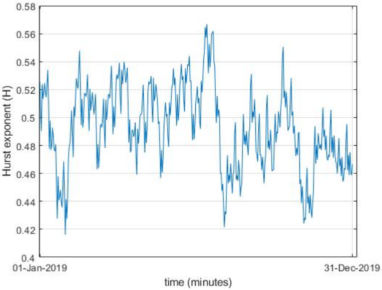
(b) Hurst exponent (2 hours)

Figura: Hurst exponent for 1 to 2 hour BTC returns, using a sliding window of 500 datapoints. Period: January 1, 2019 – December 31, 2019

Empirical findings



(a) Hurst exponent (11 hours)



(b) Hurst exponent (12 hours)

Figura: Hurst exponent for 11 to 12 hour BTC returns, using a sliding window of 500 datapoints. Period: January 1, 2019 – December 31, 2019

Theoretical framework

- The upshot previously produced by the empirical findings, namely $H = 0.5$, reveals that the process X_t evolves according to an *Arithmetic Brownian motion* (ABM)

$$\mathrm{d}X_t = \left(\mu - \frac{\sigma^2}{2}\right) \underset{.}{\mathrm{t}} + \sigma \underset{.}{\mathrm{W}}_t \,, \tag{1}$$

  with $\mu \in \mathbb{R}^+$, $\sigma > 0$, and W_t is a Wiener process, for $t \in [0, T]$.

- This is justified by observing that, from the empirical point of view, we work with $X_{t-u} = \ln(S_t) - \ln(S_u)$, for all $0 \leq u < t \leq T$, being X_{t-u} the instantaneous return associated to the time increment $t - u$. An application of Ito's Lemma ensures that the BTC price dynamics can be easily written as

$$\frac{\mathrm{d}S_t}{S_t} = \left(\mu - \frac{\sigma^2}{2}\right) \underset{.}{\mathrm{t}} + \sigma \mathrm{d}W_t, \; t \in [0, T] \,. \tag{2}$$

Forecasting BTC price by Deep Learning models

- Neural Networks (NNs) are universal function approximators
- Deep learning algorithms are data-driven models able to catch in depth features
- Recurrent Neural Networks (RNNs) allow to preserve sequential relations among the data, thanks to feed-forward and recurrent connections such that a memory is formed by construction.
- We opt for a RNNs with a Long Short Term Memory (LSTM) architecture to avoid vanishing gradient problems in forecastig the BTC price

The trading strategy

Idea

We want to experiment a *profit-oriented* strategy at a fixed time, say t_0, in which the trader bets on BTC price changes, entering a foreordained number of financial contracts, either in short or long positions. The latter depend on the profit margin the trader would like to establish, in terms of *spread* between the predicted BTC price and a given benchmark.

Contract for Difference

The best-suited financial tool is given by the so called *Contract for Difference* (CfD).

A CfD is a derivative on a financial asset. It provides that two parties agree to exchange financial flow stemming from the differential between the prices of an underlying at the beginning time of the contract and at the time of its closing. Therefore, CfDs operate on the price differences, implying gain or loss according to the difference between the purchase price and the sale price of the underlying.

The trading strategy

The touchy point is to define an appropriate benchmark. Let K be the (arranged) price agreed between the parties in the CfD, to be compared with the underlying.

In t_0 we design a decision set $D := \{d_t \, : \, t \in [0, T]\}$ such that a given rule-of-thumb is satisfied, starting from $K = S_t$. In such a case, neither a long nor a short position on CfD would guarantee speculation.

This form of indifference is represented by constructing a time-dependent benchmark, say $\mathcal{B}(t)$, that takes into account the market parameters (e.g., BTC price volatility level). Therefore, the investor must define in t_0 the position to handle in order to always reach $|S_t - K|$ as margin.

The trading strategy

Typically, this translates in keeping a short position, when the downside risk is potentially unlimited. Indeed, it is straightforward to see that

$$K > S_t, \text{ or equivalently, } \max(K - S_t, 0) > 0 \ .$$

Vice-versa, if $K < S_t$, then

$$K - S_t < 0, \text{ or equivalently, } \max(K - S_t, 0) = 0 \ .$$

This means that no profit is obtained by assuming short positions, thus a change of position towards the CfD is needed.

The trading strategy

Summing up, we create $\mathcal{B}(t)$ by means of a *synthetic derivative* with payoff $H_T = \max(K - S_T, 0)$ so that

- on $\mathcal{B}(t)$, we have $K = S_t$, that is, no decision is taken;
- under $\mathcal{B}(t)$, we have $K > S_t$ so $d_t =$ short position;
- above $\mathcal{B}(t)$, we have $K < S_t$ so $d_t =$ long position.

The trading strategy

The boundary $\mathcal{B}(\tau)$, as a function of the time to expiration $\tau = T - t$ and in absence of continuous dividend yields δ, coincides with the (unique) solution to the following weakly singular Volterra integral equation

$$\begin{aligned} K - \mathcal{B}(\tau) = Ke^{-r\tau}\Phi\left(-\frac{\log\left(\frac{\mathcal{B}(\tau)}{K}\right) + (r - \frac{\sigma^2}{2})}{\sigma\sqrt{\tau}}\right) \\ - \mathcal{B}(\tau)e^{-\delta\tau}\Phi\left(-\frac{\log\left(\frac{\mathcal{B}(\tau)}{K}\right) + (r + \frac{\sigma^2}{2})}{\sigma\sqrt{\tau}}\right) \\ + Kr\int_0^\tau e^{-r(\tau-s)}\Phi\left(-\frac{\log\left(\frac{\mathcal{B}(\tau)}{\mathcal{B}(s)}\right) + (r - \frac{\sigma^2}{2})(\tau - s)}{\sigma\sqrt{\tau - s}}\right)\mathrm{d}s \end{aligned} \tag{3}$$

The trading algorithm

Step 1. Given the time interval $[0, T]$, set the partition π.

Step 2. Construct a synthetic *American-style* derivative with strike price K, maturity T^* and payoff $H_{T^*} = \max(K - S_{T^*}, 0)$. More precisely, T^* is chosen in such a way that it coincides with the end of the trading period, while the strike price is such that $K = S_{t_1}$,.

Step 3. Determine the future trajectory $\{\bar{S}_t\}_{t \in \pi}$, by exploiting the Long Short-Term Memory (LSTM) method.

Step 4. Solve the integral equation to determine the optimal boundary $\mathcal{B}(\tau)$.
The model parameters are estimated by exploiting a maximum likelihood method.

The trading algorithm

Step 5. For all $t_i \in \pi$, define

$$Y_i := B_{t_i} - \bar{S}_{t_i} \begin{cases} > 0, & \text{implying a short position,} \\ = 0, & \text{implying a no trades are done} \\ < 0, & \text{implying a long position.} \end{cases}$$

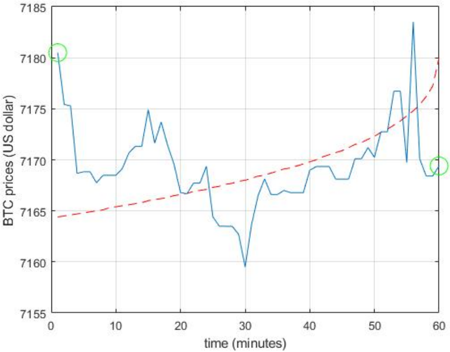
Consequently, we set h (resp. k, j) the amount of times in which Y_i is greater than (resp., equal to or smaller than) zero. Then, the strategy is composed of h long CfDs and j short CfDs.

Step 6. Evaluate the overall profit

$$G := \sum_{i=0}^{h} (B_{t_i} - \bar{S}_{t_i}) + \sum_{i=0}^{j} (\bar{S}_{t_i} - B_{t_i}) \, .$$

Step 7. (Optional) Determine the time step $t' \in \pi$ (resp., $t'' \in \pi$) where the maximum gain from short position (resp., long position) is obtained.

The trading strategy



The trading strategy over 1 hour time horizon. The forecasting starting time is December 31, 2019 – 23:59. The solid line represents the LSTM forecasting for BTC price, the dotted line is optimal boundary. The two circles indicate the maximum profit realized by the investor, both in long and short position.

Conclusions

The idea is simple, but compelling. By exploiting some Deep Learning techniques, we are able to forecast the future evolution of the BTC price process. We can set up the strategy, by taking into account a given number of suitable financial instruments (the so called Contracts for Difference), that provide profit in terms of the spread between the underlying value and the optimal frontier of a synthetic American-style derivative.

One of the key points of this work is the possibility of evaluating the BTC price through a Geometric Brownian Motion over very short time horizons. This is empirically justified by the observation that, for such time frames, the log-returns show a Hurst index equal to $H = 0.5$.

Conclusions

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Thank You for your attention