Stochastic modelling in insurance: participating policies with minimum guaranteed

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Key features of a PP

- Premiums are pooled with those of other participating policies in a specially designated participating fund
- The fund invests in a range of assets, under the control of the company
- Depending on fund's performance, benefits are paid to the policyholders

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- Guaranteed benefits: { minimum interest rate guarantee; death benefit }
- Non-guaranteed benefits: { Reversionary bonus; Terminal bonus }
- surrender benefit: only a proportion of the terminal bonuses is recognized to the policyholder

These benefits and options are liabilities to the issuer that need to be properly evaluated


Participating policy
Participating
Policy
BANK


Participating policy
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BANK


Participating
Life Insurance


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BANK


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OH NO
MY BONUSES!

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- Insurance company (solvency)

Let $T > 0$ be the maturity date of the contract

- Market value of the partecipating fund.

$$\begin{cases} \mathrm{d}A_t = A_t\big(r\mathrm{d}t + \sigma \mathrm{d}\widetilde{W}_t\big), \\ A_0 = a_0, \end{cases}$$

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- Policyholder's Reserve

$$\begin{cases} \mathrm{d}R_t = c(A_t, R_t)R_t\mathrm{d}t, \\ R_0 = \alpha\, a_0, \end{cases}$$

where $\alpha = R_0/A_0$ is the percentage of the reference portfolio financed by the policyholder

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  The insurer has a specific target for the *Buffer Ratio* $\frac{B_t}{R_t}$ (e.g.$10 - 15\%$). Surplus is credited at time $t > 0$ if

$$\ln\left(1 + \frac{B_t}{R_t}\right) = \ln\left(\frac{A_t}{R_t}\right) > \beta$$

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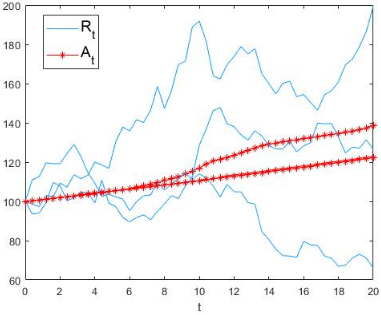
In sum

$$c(A_t, R_t) = \max\left\{\delta\left(\ln\left(\frac{A_t}{R_t}\right) - \beta\right), r^G\right\}$$

where δ is the distribution ratio (e.g. $20 - 30\%$).



Smoothing mechanism: gives the policyholder a rate of return which does not fluctuate very much

Terminal bonus

- $g(A_t, R_t) := R_t + \overbrace{\gamma\left[\alpha A_t - R_t\right]^+}^{\text{Terminal bonus payment}}, \qquad t \in [0, T]$

  where $\gamma \in (0, 1)$ is the so-called participation coefficient.

  The terminal bonus is calculated based on the policyhodelr's initial portion of assets $\alpha = \frac{R_0}{A_0}$

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- Default time $\tau^\dagger := \inf\{t \geq 0 \,:\, A_t \leq R_t\}$ (i.e. $B_{\tau^\dagger} \leq 0$)
  In the event of $\tau^\dagger < T$

$$g(A_{\tau^\dagger}, R_{\tau^\dagger}) = R_{\tau^\dagger},$$

Life insurance coverage

Let Γ_D be the residual lifetime of an individual aged η at time zero.

Survival probability

$$_{s}p_{\eta+t} = e^{-\int_0^s \mu_\eta(t+u)du}, \quad \text{for } t, s \geq 0.$$

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Death benefit: $g(A_{\Gamma_D}, R_{\Gamma_D})$

Surrender option

The policyholder is allowed to surrender the contract at any time $0 < t < T$, in which case she receives

$$(1 - k_t)g(A_t, R_t),$$

i.e the value of the policy is diminished by the surrender charge $k_t g(A_t, R_t)$, where $k_t \in [0, 1]$ is a non-increasing function of time.

The fair value of the contract at time zero is

$$V_0 = \sup_{0 \le \tau \le T} \mathrm{E}^{\mathrm{Q}}\Big[\mathbb{1}_{\{\tau < T \wedge \tau^\dagger \wedge \Gamma_D\}} e^{-r\tau}(1 - k_\tau) g(A_\tau, R_\tau) + \mathbb{1}_{\{\tau \ge T \wedge \tau^\dagger \wedge \Gamma_D\}} e^{-r(T \wedge \tau^\dagger \wedge \Gamma_D)} g(A_{T \wedge \tau^\dagger \wedge \Gamma_D}, R_{T \wedge \tau^\dagger \wedge \Gamma_D})\Big]$$

with

$$g(A, R) := \begin{cases} R & \alpha A \le R \\ R + \gamma(\alpha A - R) & \alpha A \ge R \end{cases}$$

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- To analyze the effect of the surrender charge on the optimal surrender policy (for example, find the minimal charge to eliminate the surrender incentive)

For the results, please see

M. B. Chiarolla, T. De Angelis and G. Stabile
An analytical study of participating policies with minum rate guarantee and surrender option.
Finance and Stochastics, 26, pages173-216 (2022).

Thank you!