Mathematics and finance: optimal portfolio allocation with co-jump risk

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Portfolio Asset Allocation

Portfolio Asset Allocation is the process of deciding how resources must be allocated between different possible investments

- Investors are interested in gaining as much as possible from their investment (**maximize initial wealth**) but at the same time they are concerned with the risks they have to face (**risk-aversion coefficient**)
- Theory of stochastic control to replicate mathematically the behaviour of the investor

Dynamic vs static asset allocation

- Mean-variance approach, [Markowitz H.]

static asset allocation

The initial wealth is allocated between different asset at the beginning of the period **without** the possibility of changes the allocation until the end of the period

- Continuous-time portfolio management, [Merton R.C.]

dynamic asset allocation

To consider continuous-time models for price dynamics, by allowing the ability to trade at any time, the investor can react immediately to possible **changes** in price volatility. Optimal portfolio rule are expressed in terms of the solution of HJB equation.

Dynamic Asset Allocation: incomplete market case

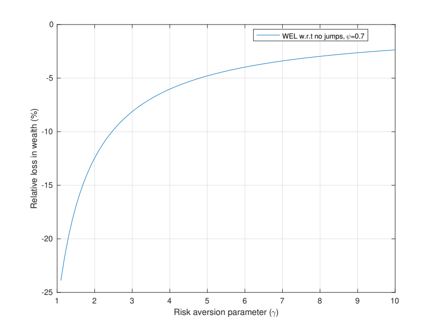
1. **Goal**: to determine the optimal proportions of wealth invested in the risk-free asset, stock and consumption-wealth ratio in a **stochastic volatility** with **co-jumps** model
2. **Recursive preferences**

Non-separable preferences are able to disentangle the investors' risk aversion from the growth rate of consumption reactivity, with respect to the interest rate trend

$$V(t, W, y) = \max_{\pi_t, C_t} \mathbb{E}\left[\int_t^{+\infty} f(C_s, V_s)ds\right]$$

Optimal portfolio allocation: incomplete market case

$$\pi_t^* = \left(\frac{\mu - r}{\gamma} - \frac{H\sigma\rho F_t}{\gamma} + \frac{\lambda J e^{-HF_t\xi}}{\gamma} \right) y_t \qquad \frac{C_t}{W_t} = \beta^{\psi} \exp\{-(F_t y_t + G_t)\}$$



Dynamic Asset Allocation: complete market case

1. **Goal**: to determine the optimal proportion of wealth invested in risk-less asset, stock and **derivatives**
2. **Completeness of market**: Adding derivative contracts to complete the market
3. **CRRA** utility function: no consumption

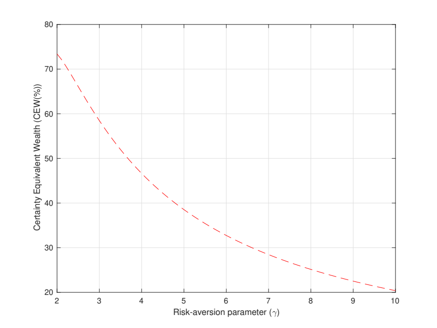
Power utility restrict the EIS to be the reciprocal of the risk-aversion coefficient: only one parameter governs both investor risk aversion and EIS

$$V(t, W, y) := \max_{\pi_t \in [0,T]} \mathbb{E}_t \left[\frac{W_T^{1-\gamma}}{1-\gamma} \right]$$

Optimal Portfolio Allocation: complete market case

Optimal exposures

$$\theta_t^{B*} = \left(\frac{\eta}{\gamma} + \frac{\sigma\rho F_t}{\gamma}\right) y_t \quad \theta_t^{Z*} = \left(\frac{\beta}{\gamma} + \frac{\sigma\sqrt{1-\rho^2} F_t}{\gamma}\right) y_t \quad \theta_t^{N*} = \frac{1}{J}\left[\left(\frac{\lambda}{\lambda^*}\right)^{\frac{1}{\gamma}} - 1\right] + \frac{1}{J}\left(\frac{\lambda}{\lambda^*}\right)^{\frac{1}{\gamma}}\left[e^{\frac{\xi F_t}{\gamma}} - 1\right]$$



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Thank you for your attention