Volterra integral equations in actuarial sciences: methods and models

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Volterra integral equations are used to model scientific, economic, physical, and biological phenomena. Although various methods are available for finding analytical solutions for integral equations, in most cases, finding a closed-form solution is not practical. We present some models and related numerical methods in actuarial sciences.

First, consider a linear Volterra integral equation of the second type, whose unique unknown solution is known to be Lipschitz-continuous. Using such a property, we derive a feasible, fast, and accurate numerical algorithm. An application to risk theory is considered. More in detail, in a Cramer-Lundberg model framework, using the corresponding integro-differential representation as a starting point, we prove that the ruin probability is a Lipschitz function. We apply the proposed algorithm to a wide range of claim size distributions, evaluating the ruin probability that solves the associated Volterra integral equation.

Then, we investigate the evaluation problem of variable annuities by considering guaranteed minimum maturity benefits, with either constant or path-dependent guarantees of up-and-out barrier and lookback type or guaranteed minimum accumulation benefit riders, with different forms of the surrender amount. We solve the non-standard Volterra integral equations associated with the policy valuations through a randomized trapezoidal quadrature rule combined with an interpolation technique. Such a rule improves the convergence rate for the classical trapezoidal quadrature, while the interpolation technique allows us to obtain an efficient algorithm that produces a very accurate approximation of the early exercise boundary. The method accuracy is assessed by constructing a benchmark developed in a lattice framework, characterized by a novel procedure for the lookback path-dependent guarantee, obtained through the lattice convergence properties.